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18MAT21

Second Semester B.E. Degree Examination, Feb./Mar.2022 Advanced Calculus and Numerical Methods

Time: 3 hrs.

Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

- 1 a. Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at the point $(2, -1, 2)$. (06 Marks)
- b. Find the divergence and curl of the vector \vec{F} if $\vec{F} = \nabla(x^3 + y^3 + z^3 - 3xyz)$. (07 Marks)
- c. Show that $\vec{F} = (y+z)\vec{i} + (z+x)\vec{j} + (x+y)\vec{k}$ is irrotational and also find a scalar function ϕ such that $\vec{F} = \nabla\phi$. (07 Marks)

OR

- 2 a. Verify Green's theorem for $\int_C (xy + y^2)dx + x^2dy$, where C is the bounded by $y = x$ and $y = x^2$. (06 Marks)
- b. Using Stoke's theorem, evaluate $\int_C xydx + xy^2dy$, where C is the square in the x-y plane with vertices $(1, 0)(-1, 0)(0, 1)(0, -1)$. (07 Marks)
- c. Using Gauss divergence theorem, evaluate $\iiint_C \vec{F} \cdot \vec{n} ds$ over the entire surface of the region above xy-plane bounded by the cone $z^2 = x^2 + y^2$ and the plane $z = 4$, where $\vec{F} = 4xz\vec{i} + xyz^2\vec{j} + 3z\vec{k}$. (07 Marks)

Module-2

- 3 a. Solve $(D^2 - 4D + 13)y = \cos 2x$, where $D = \frac{d}{dx}$. (06 Marks)
- b. Solve $(D^2 - 2D + 1)y = \frac{e^x}{x}$, by the method of variation of parameter, where $D = \frac{d}{dx}$. (07 Marks)
- c. Solve $x^3 \frac{d^3y}{dx^3} + 2x^2 \frac{d^2y}{dx^2} + 2y = 10\left(x + \frac{1}{x}\right)$. (07 Marks)

OR

- 4 a. Solve $(D - 2)^2 y = 8(e^{2x} + \sin 2x)$, where $D = \frac{d}{dx}$. (06 Marks)
- b. Solve $(1+x)^2 \frac{d^2y}{dx^2} + (1+x) \frac{dy}{dx} + y = \sin 2[\log(1+x)]$. (07 Marks)

- c. The differential equation of the displacement $x(t)$ of a spring fixed at the upper end and a weight at its lower end is given by $10 \frac{d^2x}{dt^2} + \frac{dx}{dt} + 200x = 0$. The weight is pulled down 0.25 cm, below the equilibrium position and then released. Find the expression for the displacement of the weight from its equilibrium position at any time t during its first upward motion. (07 Marks)

Module-3

- 5 a. Form the partial differential equation by eliminating the arbitrary constants form, $(x-a)^2 + (y-b)^2 + z^2 = C^2$. (06 Marks)
- b. Solve $\frac{\partial^2 z}{\partial x \partial y} = \sin x \sin y$ for which $\frac{\partial z}{\partial y} = -2 \sin y$ when $x = 0$ and $z = 0$ if y is an odd multiple of $\frac{\pi}{2}$. (07 Marks)
- c. Derive one-dimensional heat equation in the standard form. (07 Marks)

OR

- 6 a. Form the partial differential equation by eliminating the arbitrary function from $z = f(x+ct) + g(x-ct)$. (06 Marks)
- b. Solve $(y-z)p + (z-x)q = (x-y)$. (07 Marks)
- c. Solve one dimensional wave equation, using the method of separation of variables. (07 Marks)

Module-4

- 7 a. Test for the convergence or divergence of the series $\sum_{n=1}^{\infty} \frac{n!}{(n^n)^2}$. (06 Marks)
- b. Solve Bessel's differential equation leading to $J_n(x)$. (07 Marks)
- c. Express $x^4 - 2x^3 + 3x^2 - 4x + 5$ in terms of Legendre polynomial. (07 Marks)

OR

- 8 a. Discuss the nature of the series, $\frac{1}{2} + \left(\frac{2}{3}\right)x + \left(\frac{3}{4}\right)^2 x^2 + \left(\frac{4}{5}\right)^3 x^3 + \dots$. (06 Marks)
- b. With usual notation, show that
- (i) $J_{\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \sin x$
- (ii) $J_{-\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \cos x$ (07 Marks)
- c. Use Rodrigues formula to show that $P_4(\cos \theta) = \frac{1}{64} [35 \cos 4\theta + 20 \cos 2\theta + 9]$. (07 Marks)

Module-5

- 9 a. Find a real root of the equation $\cos x - 3x + 1 = 0$, correct to 3 decimal places using regula falsi method. (06 Marks)
- b. Use an appropriate interpolation formula to compute $f(42)$ using the following data:

x	40	50	60	70	80	90
$f(x)$	184	204	226	250	276	304

(07 Marks)

- c. Evaluate $\int_4^{5.2} \log x dx$ by using Weddle's rule, divided into six equal parts. (07 Marks)

OR

- 10 a. Find a real root of the equation, $x \sin x + \cos x = 0$ near $x = \pi$, correct to four decimal places. Using Newton-Raphson method. (06 Marks)
- b. Find $f(9)$ from the data by Newton's divided difference formula. (07 Marks)

x	5	7	11	13	17
f(x)	150	392	1452	2366	5202

- c. By using Simpson's $\frac{1}{3}$ rule $\int_0^1 \frac{dx}{1+x^2}$ dividing interval $(0, 1)$ into six equal parts and hence find approximate value of π . (07 Marks)
